Analytical and FEM Magnetic optimization of a limited motion actuator for automotive application

C. Gutfrind1,2, X. Jannot1, J.C. Vannier1, P. Vidal1, D. Sadarnac1

Abstract — The present work proposes a development of a limited motion actuator dedicated to the air flow regulation of an internal combustion engine. A mixed analytical-FEM optimal design methodology is presented. An analytical study of a bipolar permanent magnet actuator has been done with a first geometrical optimization under constraint by a Genetic Algorithm. Then, the analytical study is coupled with Matlab® and FEMM freeware in order to integrate a finite element method and a second optimization with a Direct Search Algorithm. The actuator performance is specified for a small volume with a maximized torque on an angular range from 0 to 100°.

Index Terms — Actuator, limited motion, electrical machine, optimization, modeling, finite element.

I. INTRODUCTION AND CONTEXT

With the recent European Standard [1] being applied to car manufacturers in order to reduce greenhouse gases and polluting emissions of novel vehicles, new conceptions of internal combustion engines need a new regulation management of air flow as described in [2] and [3].

After a state of the art on electromechanical actuators that manage the air flow and with [4], several electrical structures such as the electronic throttle control valve or the exhaust gas recycling valve have been identified and analysed. There are two main electric topologies that produce rotary movements: the first one is an indirect drive composed of a DC motor and a reduction gear, the second one is a direct drive with a torque motor. For the present application, a direct drive actuator has been chosen in order to avoid non linear behaviour of the gear set.

To study this actuator, a simple analytical model is developed to parameter its geometry and to include the calculation of magnetic flux density and torque. But, in this analytical model, the cogging torque is not described while it should be necessary taking to account this non-linear effect in order to obtain a constant torque on a 100° range.

The association of a FEM model and the analytical model with an optimization sequence should be an interesting method to define the actuator model.

II. ACTUATOR DEVELOPMENT METHOD

A. Actuator topology

The actuator is composed of two coils, two "tile" form magnets joined to the rotor and a magnetic circuit or stator. Magnets and coils create a radial magnetic flux that crosses the rotary axis of the rotor as in Fig. 1. The variation of magnetic flux linked by the coils creates a torque and this performance is studied.

In general, as in [5], the torque has been defined by a coefficient that depends of actuator topology, , electrical loading in coils, , rotor volume and magnetic flux density in relation with magnets and stator geometry as:

\[ T = k \cdot A_L \cdot V_e \cdot B_{max} \] (1)

Fig. 1. Bipolar permanent magnet actuator topology

B. Optimization flowchart

The analytical study defines the sizing parameters of the actuator geometry and the electromagnetic characteristics. The optimization objective is to calculate the optimal values of these parameters in order to maximize the torque according to the constraints.

With this model, the main difficulty is to obtain a constant torque on an angular range from 0 to 100° while the tooth design influences the cogging and final torque. The interaction between slot geometry and magnet geometry is essential in actuator development.

FEMM is a Magnetic Finite Element Method freeware developed by David Meeker [6]. Geometrical and physical descriptions of the actuator have been developed with Matlab®.

With the analytical model, the first sequence of an optimization has been realized with a Genetic Algorithm (GA).

In this article, at first, analytical and FEM models are described with defined constraints. The optimization problems and results are presented in a second part.
with a Direct Search Algorithm (DSA) refines the geometrical design in order to integrate the cogging torque effect. The optimization flowchart is resumed in Fig. 2.

Fig. 2. Flowchart procedure of optimization.

III. ANALYTICAL MODEL

To obtain the flux and torque expressions, the air gap magnetic flux density is defined. A cylindrical reference \((r, \alpha)\) is fixed to the stator in order to visualize the magnetic flux density. And a moving cylindrical reference \((r, \theta)\) is fixed to the rotor to visualize the actuator torque. \(\theta\) is a rotational angle of the rotor in regard of \(\alpha=0^\circ\).

A. Air gap magnetic flux density calculation

The relation of magnetomotive force is \(f_{\text{mm}} = n \cdot I\) where \(n\) is the quantity of turns in slot pair and \(I\) the current in coil. In Fig. 3, Ampere’s law on \(C\) path is applied, flux crosses two times air gap thickness \(e\) and two times magnet thickness \(e_a\):

\[
2 \cdot (H_a \cdot e_a + H_e \cdot e) = f_{\text{mm}} = nI
\]

Where \(H_a\) is the magnet field and \(H_e\) the air gap field.

Magnet field is given by:

\[
H_a = \frac{B_a - B_e}{\mu_0 \cdot \mu_r}
\]  

With \(\mu_0\) the vacuum permeability, \(\mu_r\) the magnet relative permeability and \(B_e\) the remanent flux density of magnet and \(B_a\) the magnetic flux density in magnet.

And magnetic field in air gap is defined by:

\[
H_e = \frac{B_e}{\mu_0}
\]  

With \(B_e\) the air gap magnetic flux density in air gap.

Gauss’s law links the flux crossing the air gap section \(S_e\) and magnet section \(S_a\):

\[
B_a \cdot S_a = B_e \cdot S_e \quad \text{then} \quad B_a = B_e \cdot \frac{S_e}{S_a} = B_e \cdot \frac{R_e}{R_a} = B_e \cdot k_{ea}
\]

Where \(k_{ea}\) is the ratio between the average radius of the air gap \(R_e\) and the magnet average radius \(R_a\).

The air gap magnetic flux density is given by:

\[
B_e(\theta) = \frac{B_e \cdot e_a}{k_{ea} \cdot e_a + \mu_r \cdot e} \cdot \beta(\theta) + \frac{\mu_0 \cdot n \cdot I}{2 \cdot (e + k_{ea} \cdot \frac{e}{\mu_r})} \cdot \delta(\theta)
\]

And finally:

\[
B_e(\theta) = B_e \cdot \beta(\theta) + B_a \cdot \delta(\theta)
\]

Functions \(\beta\) and \(\delta\) equal -1 or 1 according to the position of observation in the air gap as shown in Fig. 4. The magnets flux density, \(B_{ma}\), depends on the rotor position, the curve is variable according to \(\theta\). The stator current reaction, \(B_{ni}\), is fixed whatever the rotor position. The resulting flux density in the air gap, \(B_e\), is the sum of both. The integration window allows the calculation of the flux under one pole.

Fig. 4. Air gap flux density distribution at \(\theta=180^\circ\) in \((r,\alpha)\)

B. Magnet demagnetization constraint

Permanent magnets are made of SmCo because of their heat resistance characteristic. The magnet model is simplified for the analytical study. From the equations (2), (4) and (5), the load line of the magnet is deduced:

\[
B_a = \mu_0 \cdot k_{ea} \cdot \frac{e_a}{2 \cdot e_a} \left(\frac{n \cdot I}{2 \cdot H_a} - H_a\right)
\]

Magnet minimal thickness should verify the following condition to avoid demagnetization:

\[
e_a > \frac{n \cdot I}{2 \cdot H_e}
\]
C. Excitation flux expression

The excitation flux in air gap is defined by:
\[ \Phi_e = n \cdot R \cdot L \cdot B_{L} \cdot (\pi - 2\theta) \]  

(10)

The magnet flux depends on \( \theta \), the rotor position in regard to stator as in Fig. 5.

D. Computation of the magnetic flux densities in the iron paths

The maximum magnetic flux densities in rotor and stator are calculated with the maximal flux that crosses the rotor and stator section.

Saturation flux density is calculated in rotor:
\[ B_{\text{rotor, saturation}} = \frac{\text{max}(\Phi_e)}{S_{\text{rotor}}} \]  

(11)

Saturation flux density is calculated in stator:
\[ B_{\text{stator, saturation}} = \frac{\text{max}(\Phi_x)}{S_{\text{stator}}} \]  

(12)

E. Torque definition

The magnetic co-energy is given by [7]:
\[ W_m = \sum_i \frac{1}{2} \Phi_i \cdot I_i \]  

(13)

The torque value is defined by the variation of the magnetic flux. In magnetic coenergy derivative, the part of the flux that varies with the rotor position is used as \( \theta \in [0, \pi] \).

\[ T = \left( \frac{\partial W_m}{\partial \theta} \right)_{\text{rotor}} = \frac{\partial}{\partial \theta} \left[ n \cdot I \cdot R \cdot L \cdot B_{L} \cdot (\pi - 2\theta) \right] \]  

(14)

And torque expression is given by:
\[ T = -2 \cdot n \cdot R \cdot L \cdot B_{L} \cdot I \]  

(15)

Or \( T = -2 \cdot J \cdot S_{\text{slot}} \cdot R \cdot L \cdot B_{L} \)

Where \( J \) is the current density in a slot section \( S_{\text{slot}} \) with a slot filling factor equal to 1.

Finally, for example, torque evolution according to rotor position is schematized in Fig. 6. Torque values are negative on a range from 0 to 180°.

Moreover, in the Fig. 6, torque curve shows the influence of opening angles from magnet and teeth.

F. Magnet and tooth influence on torque curve

Actuator has to supply a constant torque on a 100° range of rotation. \( A_{\text{OA}} \) and \( A_{\text{OS}} \), the opening angles of magnets and teeth, are such as Fig. 7.

The interaction of the magnet and the stator opening angle defines a null torque on angular range denoted as \( A \):
\[ A = \frac{A_{\text{OS}} - A_{\text{OA}}}{2} \]  

(16)

The torque definition at part \( B \):
\[ B = (\pi - A_{\text{OA}}) \]  

(17)

The constant torque range is defined by:
\[ T = \pi - 2 \cdot (A + B) \]  

(18)

Figs. 8 to 10 show an example of a tooth opening angle of 120° and a magnet opening angle of 170°.

The part of \( A \) results in 25° angle. The part of \( B \) results in 10° angle. The constant torque range is 110°.
In Fig. 8, the rotor turns between 0° at 25°: the torque range A means that the tooth surface $S_t$ covers all magnet surface $S_a$ and gets no variation of the magnetic flux.

![Fig. 8. Rotor displacement from 0° to 25°](image)

In Fig. 9, the rotor turns between 25° at 35°. The torque range B means that the teeth get less and less magnet flux.

![Fig. 9. Rotor displacement from 25° to 35°](image)

In Fig. 10, the rotor turns between 35° at 145°: a constant torque range is available for the application.

![Fig. 10. Rotor displacement from 35° to 145°](image)

IV. FINITE ELEMENT MODEL

In order to check and improve our analytical model, the FEMM freeware is used and coupled with Matlab® for finite element computation.

A. FEMM script definition

The FEMM actuator geometry has been defined in a Matlab® script with initial values parameters issued from the first analytical model calculation. Actuator geometry has some symmetry but the simple model allows studying it on all its section. Then, properties of the materials have been defined in every region as magnets, core, air, air gap and shaft rotor. A magnetic boundary condition is on the external radius.

The stator and rotor axis are composed of a nonlinear magnetic material with a relative permeability $\mu_r=10000$ and a saturation flux density of 1.8T.

A magnetostatic physic formulation is used to compute the actuator performances.

The final geometry is presented in Fig. 11.

![Fig. 11. Geometrical and meshed model with FEMM](image)

B. Torque computation

The Matlab® script develops an automatic process of geometrical construction and torque resolution. The computation of torque can be applied at every rotor position and torque curve can be laid out as in Fig. 14.

The torque is computed over a 100° range, with 10 computation steps.

The torque on the 100° range is calculated in order that their values are included in a 5% range of maximum torque. A new parameter is added to define the beginning of constant torque range.

V. OPTIMIZATION PROBLEM AND RESULT

A GA has been used to optimize the model parameters of the analytical model. Two optimization scripts have been written with Matlab® to calculate the optimal geometry of actuator.

The objective is to maximize the torque with a current density of 5 A/mm² fixed in the slot section with 150 turns at 20°C temperature.

Constraints are:
- a space requirement of 40mm diameter and 40mm height,
- stator and rotor flux density should be lower than 1.8T,
- magnet demagnetization in (9),
- constant torque on a range of 100° minimum.

Optimization variable parameters are:
- magnet geometry : its thickness $e_a$ and opening angle $AOA$,
- magnet remanent flux density $B_r$ with a relative permeability $\mu_a$ is equal at 1.03,
- air gap thickness $e$ and its opening angle $AOS$,
- stator thickness $es$ and external radius of rotor $Ra$,
- beginning torque angle range for FEM optimization.

Eight variables parameters are optimized. Every optimized variable finds its value in a range limited by a minimum value and a maximum value, see Table I.
Starting from an initial point that respects the constraints, the algorithm begins the optimization.

A. First optimization results with the analytical model

For optimization, a Personal Computer has been used for this study. The performance is a 1.8GHz duo core Intel processor with 1 Go Ram memory.

GA is a sequence of a selection, mutation and crossover of individuals from a population as described in [8], [9], [10]. An individual is a parameter vector composed by genes. A gene is a variable parameter of actuator. Elitism is used to conserve the best individuals of a generation. This strategy copies the best individual from generation \( n-1 \) into generation \( n \). GA converges to the global best individual who defines the optimized actuator.

The optimal values obtained with the GA have been found at the 707\(^{th}\) generation, and computation took 17h45min. If torque doesn’t vary anymore after 150 generations, GA stops the optimization. Global optimum is considered reached.

The parameter evolution is in accordance with the orientations defined by the air gap flux density and torque equations. The torque maximization leads to:
- minimize the air gap, and optimize the magnet, rotor and stator thickness,
- minimize tooth opening angle and maximize magnet opening angle,
- maximize remanent flux density of magnets.

The results of the analytical model are given in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Min. limit</th>
<th>Initial value</th>
<th>Max. limit</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air gap thickness ( e )</td>
<td>mm</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Stator thickness ( es )</td>
<td>mm</td>
<td>2</td>
<td>4</td>
<td>10</td>
<td>3.256</td>
</tr>
<tr>
<td>Opening stator angle ( AOS )</td>
<td>°deg</td>
<td>100</td>
<td>120</td>
<td>170</td>
<td>110.12</td>
</tr>
<tr>
<td>Magnet thickness ( ea )</td>
<td>mm</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>3.11</td>
</tr>
<tr>
<td>Opening magnet angle ( AOA )</td>
<td>°deg</td>
<td>100</td>
<td>170</td>
<td>170</td>
<td>168.30</td>
</tr>
<tr>
<td>Remanent flux density ( Br )</td>
<td>T</td>
<td>0.8</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Rotor radius ( Ra )</td>
<td>mm</td>
<td>1</td>
<td>4</td>
<td>8</td>
<td>3.26</td>
</tr>
</tbody>
</table>

Fig. 6 presents the torque curve at 5A/mm\(^2\) in slot section. The maximized torque is 0.370N.m and the range of constant torque is 100°, from 40° to 140°.

B. Second optimization result with FEMM model

For the second optimization with DSA, the optimized parameter values provided by the first optimization model are used.

Optimization results of parameters are obtained with the same computer at the 68\(^{th}\) iteration and after a computing time of 1h50min.

The maximized torque is 0.355N.m and the range of constant torque is 100°, from 40° to 140°. Parameters values are illustrated in Table II.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Min. limit</th>
<th>Initial value</th>
<th>Max. limit</th>
<th>Optimal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air gap thickness ( e )</td>
<td>mm</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Stator thickness ( es )</td>
<td>mm</td>
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<td>3.256</td>
<td>10</td>
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<td>Opening stator angle ( AOS )</td>
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<td>170</td>
<td>170</td>
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</tr>
<tr>
<td>Magnet thickness ( ea )</td>
<td>mm</td>
<td>1</td>
<td>3.11</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Opening magnet angle ( AOA )</td>
<td>°deg</td>
<td>100</td>
<td>170</td>
<td>170</td>
<td>168.30</td>
</tr>
<tr>
<td>Remanent flux density ( Br )</td>
<td>T</td>
<td>0.8</td>
<td>0.95</td>
<td>0.95</td>
<td>0.95</td>
</tr>
<tr>
<td>Rotor radius ( Ra )</td>
<td>mm</td>
<td>1</td>
<td>3.26</td>
<td>8</td>
<td>3.26</td>
</tr>
</tbody>
</table>

In Fig. 12, stator and rotor flux density distribution show that the actuator geometry respects saturation constraints.

The most constraining position for magnetic circuit is the combined position of magnets and coil flux. Then, the saturation level of circuit is reached. For intermediate positions of rotor, the magnetic flux density in the pole is under the saturation level.

Fig. 12. Rotor and stator flux density distribution

In Fig. 13, torque curves obtained by FEM show a cogging torque of actuator when \( J=0A/mm^2 \) in a slot section and a constant torque when \( J=5A/mm^2 \). The third curve is a torque of 5A/mm\(^2\) without the cogging torque effect (curve in continuous line) in order to compare the analytical and FEM torque curves.

Fig. 13. Torques evolutions curves
In actuator application, the available torque goes from 40° position to 140° position. The cogging torque can be considered like an offset torque on rotor position that the analytical model cannot define. FEM model analysis is necessary to refine the actuator model and to obtain a more accurate solution.

C. Validation of the analytical model

In Fig. 14, analytical and FEM torque curves are compared and this shows:
- the good behaviour of the analytical model for the computation of the constant torque value,
- the interest of this optimization method to consider the difference between a “rectangular curve” of analytical model and the FEM curve.

Another nonlinear effect at the 100° position is a flux density concentration on the tooth pole. If this saturation level area is a constraint, the actuator will not globally be optimized and maximal torque value will be less important. This particularity can be difficult to integrate in the analytical model. The space mapping method should be interesting to apply on this local point.

Comparison results of the analytical and FEM model with the DSA parameters values are detailed in Table III.

![Fig. 14. Torques curves comparison between analytical model and FEM model at J=5A/mm², for the optimal FEM solution](image)

VI. CONCLUSION

A. Results from coupling analytical and FEMM model

A first method was to calculate the optimal values parameters and to verify the electromagnetic model with FEM software. Now, the automated optimization with the coupling of analytical and FEMM model promise a good method to define a geometrical model of a permanent magnet machine according to given requirements.

The only disadvantage is the time resolution of Genetic Algorithm. It needs more time to solve the optimization problem than SQP or DSA. Then, the methodology has many advantages as:

- GA is a robust optimization procedure that converges on a global solution avoiding the risk of a local solution,
- when the analytical model doesn’t consider the cogging torque and nonlinear effects of iron material, the method is interesting to couple the analytical and FEM models,
- a saturation level is more finest with FEM.

B. Future improvement of model

After an electromagnetic approach, the machine design can be improved in order to refine the actuator as:
- dynamic and driving simulation, time response and current are specified and it is interesting to integrate these requirements in order to influence the geometrical design,
- thermal simulation to define the heat resistance of the actuator,
- cost constraints of materials and manufacturing.

VII. REFERENCES

[9] X. Jannot is working towards the Ph.D. degree in Electrical Engineering with the “Department of Electrical Power Systems” in Supelec. His main research interest includes multidisciplinary design of permanent magnet synchronous machine with their power electronic supply.

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Pierre Vidal is Professor in “Department of Electrical Power Systems” in the Ecole Supérieure de l’électricité (Supelec) in France. His research interest concerns the electrical system conception as electronic converters, electric machines and with associating mechanical system (gearbox, movement transformation).

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